Automated Verification
Mid-Semester Exam (2006)

Max Marks: 28, Weightage: 20%, Time: 2 hours

1. Consider the ω-language over the alphabet $\{a, b, c\}$ containing all infinite words in which every a is eventually followed by a b, and every b is eventually followed by a c. Give a Büchi automaton which accepts the language. *(2)*

2. Recall that McNaughton's theorem states that deterministic Muller automata have the same expressive power as Büchi automata. *(3)*

   (a) Describe briefly the route to complementation of Büchi automata taken by Safra's construction.

   (b) Show how McNaughton's theorem can be proved using Safra's construction.

3. The *kernel* of an ω-language is the set of all ultimately periodic words in it (i.e. words of the form $u.v^\omega$.) Show that two ω-regular languages are the same if and only if their kernels are the same. *(4)*

4. Which of the following temporal logic identities are correct? Justify your answer. *(2)*

   (a) $\Box(\varphi \vee \psi) \equiv \Box(\varphi) \vee \Box(\psi)$

   (b) $\Diamond(\varphi \wedge \psi) \equiv \Diamond(\varphi) \wedge \Diamond(\psi)$

5. Describe the Vardi-Wolper automaton for the formula $((p \vee \neg p)Up)$. *(4)*

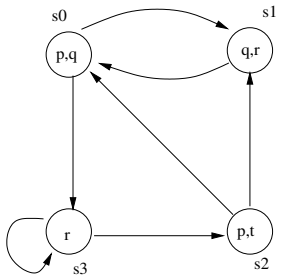
6. LTL can be interpreted over $finite$ words over the alphabet 2^P in a natural way. Describe a way of interpreting LTL over these finite models (i.e. give the formal semantics).

   Describe how you can modify the Vardi-Wolper construction to obtain a classical NFA $\mathcal{A}_\varphi$ which accepts precisely the set of finite models of an LTL formula φ. *(4)*

7. Draw a Venn diagram to show the relationship between the classes below. Give an example of a language in *each region* of your diagram *(4)*

   - LTL: the class of all languages over $\{a, b\}$ which can be described by LTL formulas
   - ω-REG: the class of ω-regular languages over $\{a, b\}$.
   - Det-ω-REG: the class of deterministic ω-regular languages over $\{a, b\}$ (i.e. those accepted by deterministic Büchi automata).

8. Consider the transition system $\mathcal{T}$ given below.



In which states of $\mathcal{T}$ does the CTL formula $AG(EF(p \vee r))$ hold? (*3*)

9. Give brief answers to the following questions: (*2*)

   (a) True or False: To check if a Büchi automaton has a cycle containing a final state, and that this cycle is reachable from the initial state, requires time quadratic in the number of states and transitions in the automaton

   (b) Give a Monadic Second Order Logic formula which describes the language of Question 1.