**Automated Verification**

*Final Examination (2006)*

Max Marks: 37, Weightage: 40%, Time: 3 hours

1. Give a Büchi automaton that accepts the set of models of the LTL formula $pUq)Ur$. (This need not be the Vardi-Wolper automaton.) *(3)*

2. A *parity* condition was defined by Mostowski as an acceptance condition on infinite words. A parity condition is given by a function $index : Q \rightarrow \mathbb{N}$ which associates an index or "rank" with each state in $Q$. A run $\rho$ is accepting according to a parity condition iff the least rank among the states that occur infinitely often along the run is even. More precisely, $\rho$ is accepting to the parity condition *index* iff

$$\min\{index(q) \mid q \in inf(\rho)\}$$

is even.

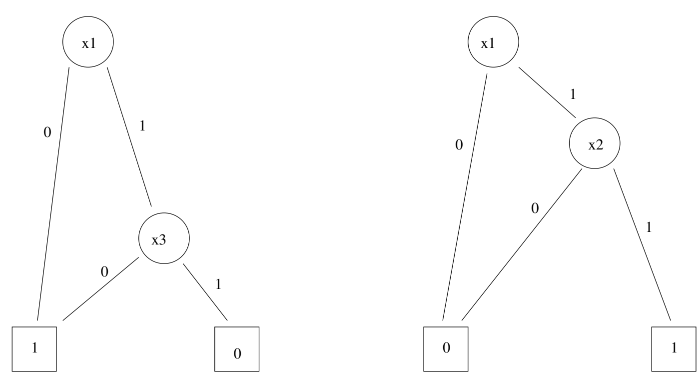
   (a) Show that parity automata (i.e. finite-state automata with a parity acceptance condition) have the same expressive power as Büchi automata. *(4)*

   (b) How will you complement a deterministic parity automaton? *(2)*

3. Run the CTL model checking algorithm for the formula $(\neg error) \Rightarrow AFheat$ on the given transition system. List the set of states which satisfy the subformulas you consider. *(4)*

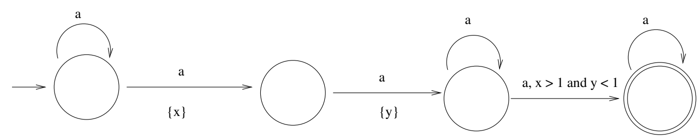
4. Give a transition system that distinguishes the CTL* formulas $EGFp$ and $EGEFp$. *(3)*

5. Construct a BDD that represents the conjunction of the boolean expressions represented below. Reduce the resulting BDD. *(4)*



6. Recall that the expression $\exists x_i f$ denotes a boolean function $g$ given by $g(b_1,\ldots,b_n) = 1$ iff there exists a boolean value $b$ for $x_i$ for which $f(b_1,\ldots,b_{i-1},b,b_{i+1},\ldots,b_n) = 1$. Give the ROBDD representing the expressions $x_1 \cdot x_2 + x_3$ and $\exists x_2(x_1 \cdot x_2 + x_3)$. Use the ordering $x_1 < x_2 < x_3$ on variables. *(2)*

7. What is the language accepted by the timed automaton below? List the "regions" corresponding to this timed automaton in the region construction. *(4)*




8. Recall that MTL extends LTL with an interval indexed until operator $U_I$. In the "pointwise" semantics, for a timed word $\sigma = (\vdash, 0)(a_1,t_1)\cdots(a_n,t_n)$ and a position $i \in \{0,\cdots,n\}$, we have

$$\sigma, i \models \varphi_1 U_I \varphi_2 \text{ iff } \exists k \geq i \;:\; \sigma, k \models \varphi_2, t_k - t_i \in I, \text{ and } \forall j \;:\; i \leq j < k \;:\; \sigma, j \models \varphi_1.$$

In the "continuous" semantics, we have for $\sigma$ and a real-valued time point $t$ with $0 \leq t \leq t_n$ :

$$\sigma, t \models \varphi_1 U_I \varphi_2 \text{ iff } \exists t' \geq t \;:\; \sigma, t' \models \varphi_2, t' - t \in I, \text{ and } \forall t'' \;:\; t \leq t'' < t' \;:\; \sigma, t'' \models \varphi_1.$$

Give a formula which is the pointwise interpretation expresses the same property as the formula $\Box(a \Rightarrow \Diamond\Diamond_{[1,1]} b)$ in the continuous semantics. *(2)*

9. Let us represent a finite timed word using delays instead of absolute timestamps as follows. We write $\sigma = \;\; \vdash d_1 a_1 d_2 a_2 \cdots d_n a_n$, where each $d_i$ is a positive real, and each $a_i \in \sum$, instead of $(\vdash, 0)(a_1, d_1)(a_2, d_1 + d_2), \cdots, (a_n, d_1 + d_2 + \cdots + d_n)$. Given finite timed words $u$ and $v$ in this representation, we can consider the "periodic" sequence of timed words $\langle \sigma_i \rangle$ induced by them, given by $\sigma_i = u \cdot v^i$ for each $i$.

   We can now define a notion of when such a periodic sequence $\langle \sigma_i \rangle$ *ultimately satisfies* a pointwise MTL formula $\varphi$ : that is, if there exists a $k$ such that for all $i \geq k$, we have $\sigma_i, 0 \models \varphi$. Similarly, we say $\langle \sigma_i \rangle$ *ultimately does not satisfy* $\varphi$ if there exists a $k$ such that for all $i \geq k$, we have $\sigma_i, 0 \not\models \varphi$. Pavithra shows the following result: let $\varphi$ be any MTL formula of *granularity* $p$ (i.e. $p$ is of the form $1/m$ and every interval in $\varphi$ has bounds which are integral multiples of $p$), and let $\langle \sigma_i \rangle$ be given by $u$ and $v$ with the time length of $v$ equal to an integral multiple of $p$; Then $\varphi$ is either ultimately satisfied in $\langle \sigma_i \rangle$ or is ultimately unsatisfied in $\langle \sigma_i \rangle$.

   Use this result to argue that the timed language of "even $b$'s" (i.e. all timed words over the alphabet $\{a, b\}$ which have an even number of $b$'s) is *not* definable by any MTL formula in the pointwise semantics. *(2)*

10. Draw a Venn diagram depicting the classes of timed languages describable by MTL in the pointwise semantics, timed automata, and all possible timed languages. These classes are all over the alphabet $\{a, b, c\}$. Demonstrate a timed language in each region of your diagram. *(3)*

11. Give brief answers to the following questions based on seminars done in class: *(4)*

    (a) What are "counter-free" automata. What is the connection between LTL and counter-free automata?

    (b) What symbolic representation is convenient to use in bounded model checking?

    (c) Give a property expressible in LTL but not in CTL, which was covered during this seminar.

    (d) What are "linear" hybrid systems?