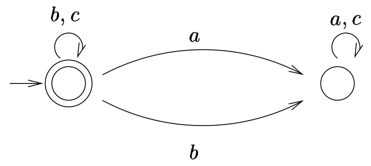
Automated Verification

Assignment 2

(Due on Tue 14th Feb 2006)

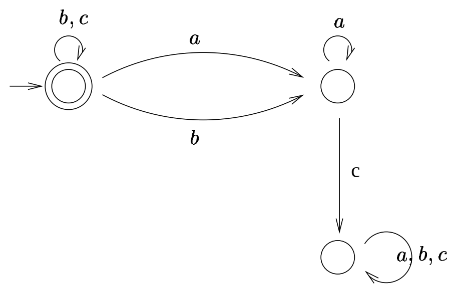
1. Describe the language accepted by the following Büchi automaton:



2. Prove or disprove: every finite language of ω-words is ω-regular.

3. Prove that the language $L = \{a^i b^i \mid i \geq 0\}^\omega$ is not ω-regular.

4. Consider the Büchi automaton $\mathcal{A}$ below:



   (a) Describe the language accepted by $\mathcal{A}$.

   (b) Give a Büchi automaton that accepts its complement.

   (c) Prove that you cannot give a *deterministic* Büchi automaton that accepts the complement of $L(\mathcal{A})$.

5. A *generalised* Büchi acceptance condition is given by a set $\mathcal{F} = \{F_1, \ldots, F_k\}$, with each $F_i \subseteq Q$, where Q is the set of states of the automaton. A run ρ is accepting according to this condition iff $\mathit{inf}(\rho) \cap F_i \neq \emptyset$ for each $i \in \{1, \ldots, k\}$. Argue that generalized Büchi automata have the same power as Büchi automata.

6. Consider the ω-language L over $\Sigma = \{a, b\}$, comprising of all ω-words which contain infinitely many a's *and* b's.

   (a) Give a Muller automaton which accepts L.

   (b) Give a Buchi automaton which accepts L.

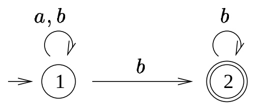
7. A *Rabin* condition is given by a set of pairs $\mathcal{R} = \{(R_1, G_1), \ldots, (R_k, G_k)\}$, with each $R_i, G_i \subseteq Q$. A run ρ is accepting according to the Rabin condition above, iff there is a pair (R_i, G_i) such that $\mathit{inf}(\rho) \cap R_i = \emptyset$ *and* $\mathit{inf}(\rho) \cap G_i \neq \emptyset$. i.e. the "red" states (those in R_i) are seen only finitely often, while the "green" states (those in G_i) are seen infinitely often.

   (a) Show how to simulate a Büchi automaton by a Rabin automaton.

   (b) Show how to simulate a Rabin automaton by a Büchi automaton.

8. Recall the congruence $\sim_{\mathcal{A}}$ defined in class for a given Büchi automaton $\mathcal{A} = (Q, s, \rightarrow, F)$.

   (a) Describe the equivalence classes of this relation for the Büchi automaton below.



   (b) Give a procedure to compute the $\sim_{\mathcal{A}}$ equivalence classes for a Büchi automaton $\mathcal{A}$ in general. (Hint: use the sets $W^F_{qq'}$ and $W_{qq'}$ discussed in class.)