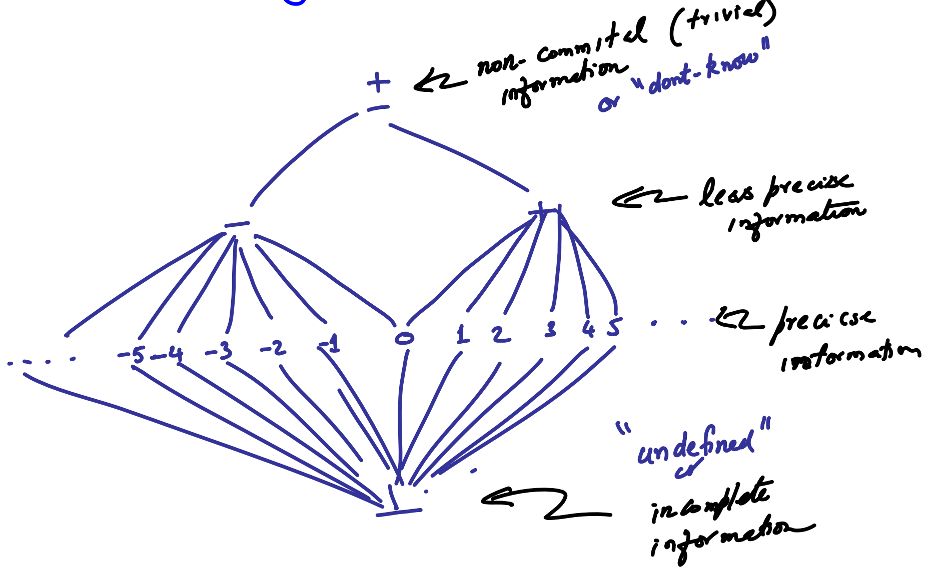
Assignment Questions :

1 Recall the "signs" abstract domain

This domain keeps track of the sign of an integer



1) Design an abstract domain with finite height that keeps track of the **remainder when divided by 3** (mod 3)

2) What are α & γ? Prove that they form a Galois connection

3) What are the abstract interpretations of the usual operators:

a) assignments

b) conditionals

2. Recall the definition of the Narrow Operator of $\mathcal{A}: \langle A, \omega, \leq, \top, \bot, I \rangle$

$T(X) = X \cup I(X)$

Narrow operator : Δ

A technique to enforce convergence of decreasing sequences...

let $s_0 \geq s_1 \geq s_2 \; \ldots\ldots$

$\Delta : A \longrightarrow A$ is a narrow operator if it satisfies:

1. $\forall \; s, s' \in A$. $\; s \geq s' \Rightarrow s \geq s \Delta s' \geq s'$

2. for any decreasing sequence $s_0 \geq I(s_0) \geq I^2(s_0) \ldots \ldots$

the sequence $t_0 = s_0$, $\; t_n = t_{n-1} \triangledown I(t_n)$

i.e. $t_0 \geq t_1 \geq t_2 \ldots\ldots$, converge

Suppose you start the decreasing sequence

$s_0 \geq I(s_0) \ldots\ldots$

from a widened fixpoint that is above the least fix point of a system

i.e.. $s_0 \geq lfp(T)$

PROVE THAT THE FIX POINT of:

$t_0 \geq t_1 \geq t_2 \ldots$

IS A SOUND fixpoint (i.e... it is "above" the least fix point)

i.e... $lfp(t_0, t_1, t_2, \ldots) \geq lfp(T)$

HINT: SEE 9.3.1 IN COUSOT-COUSOTS' PAPER

3. Consider two variants of the narrow operator

V1: $[i, j] \Delta [k, l] =$
[if $i = -\infty$ then k else $\min(i, k)$ fi,
if $j = \infty$ then l else $\max(j, l)$]

V2: $[i, j] \Delta [k, l] = [\min(i, k), \max(j, l)]$

For both these variants:
Do they satisfy the <u>two</u> conditions for a <u>narrow</u> operator (ie.. soundness & convergence)?