DAA 2006: Final Exam

Q1.[6] Show that any instance of SAT (i.e., a boolean formula in conjuctive normal form) can be expressed as an instance of Inter Linear Programming.

Q2.[6] Describe a simple randomized contraction algorithm to determine a min-cut in a connected undirected multigraph. Analyze its error probability.

Q3.[6] Consider Linear Programming in 3 dimesions: we are given n half-spaces $h_1, \cdots, h_n$, in 3 variables and a linear objective function in these 3 variables. We want to find the point in $h_1 \cap h_2 \cap \cdots \cap h_n$ that achieves the maximum value of the objective function. Let us try ot develop an incremental algorithm for it. For each i, let $C_i = h_1 \cap h_2 \cap \cdots \cap h_i$ and also assume that C_i is bounded in the direction of increasing objective function.

Let v_i denote the point in C_i that maximizes the objective function. Prove the following claim:

– if $v_i \in h_{i+1}$, then $v_{i+1} = v_i$.

– if $v_i \notin h_{i+1}$, then either $C_{i+1} = \phi$ or v_{i+1} belongs to the hyperplane that define h_{i+1}.

[Note: if h_{i+1} is $ax_1 + bx_2 + cx_3 \leq d$, then the hyperplane that defines h_{i+1} is $ax_1 + bx_2 + cx_3 = d$.]

Q4.[8] Show that a maximum cardinality independent set in a bipartite graph can be computed in time that is polynomial in the size of the graph.

Q5.[8] Consider the following multiprocessor shceduling problem. The input consists of n jobs: $J_1, \cdots, J_n$. Job J_i needs run-time p_i, for each $1 \leq i \leq n$. Assume each p_i to be a rational number. The jobs are to be scheduled on$m$ identical processors so as to minimize $\max_{1 \leq i \leq m} S_i$, where S_i = sum of the run-times of jobs assigned to processor i. This is an NP-hard problem.

Let us design an approximation algorithm for this problem. Let our algorithm consider the n jobs one-by-one and assign job J_k, for $1 \leq k \leq n$, to a processor which at that point is the least loaded processor (i.e., it has the least value of S_i, taking into account only the assignments of $J_1, \cdots, J_{k-1}$). Show that this algorithm has an approximation ratio of 2-1/m.

Q6.[8] Recall the 2-approximation algorithm for the Traveling Salesman Problem with triangle inequality on its costs. Improve this to a 3/2-approximation algorithm using the following hint:

The 2-approximation algorithm traversed each edge in the Minimum Spanning Tree (call it T) twice and then took short-cuts to a get a tour. Instead now, consider a minimum weight complete matching M in the subgraph induced by the odd-degree vertices in T. Such a minimum weight complete matching can be determined in polynomial time. Make adjustments on the graph $T \cup M$ to a get a good tour. Prove the 3/2 bound.

Q7.[8] Give a linear-time algorithm to determine if a text T is a cyclic rotation of itself. For example, *car* and *arc* are cyclic rotations of each other.