Assigment sheet 2

Due Date: 1 March 2006

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1. A simple polygon P (i.e., a polgon without holes) is called *star-shaped* if it contains a point q such that for any point $p \in P$, the line segment $\overline{pq}$ is contained in P. Give an algorithm whose expected running time is linear to decide whether a simple polygon is star-shaped.

2. Consider the following problem, called the *smallest enclosing disc* problem: given a set P of n points in the plane, find the smallest disc that contains all the points of P.

   Design a randomized incremental algorithm for this problem whose expected running time is $O(n)$.

3. Given an example of a set of n line segments with an order on them that makes the planar point location algorithm create a search structure of size $\Theta(n^2)$ and worst-case query time $\Theta(n)$.

4. Given a set P of n points, their *convex hull* is the smallest convex set that contains all the n points. That is, it is the intersection of all convex sets that contain the n points in P.

   – A *convex combination* of points $x_1, x_2, \cdots, x_n$ is a point of the form $s_1x_1+s_2x_2+\cdots+s_nx_n$ where each $s_i \geq 0$ and $s_1+s_2+\cdots+s_n = 1$. Using the fact that every point on a segment $\overline{ab}$ can be wirtten as $sa + (1-s)b$ for $0 \leq s \leq 1$ (as s goes from 0 to 1, we traverse the segment from the endpoint b to the endpoint a), show that every point in the convex hull of a set of n points can be written as a convex combination of the vertices of the convex hull.

   – Design a randomized incremental algorithm with expected running time $O(n \log n)$ to compute the convex hull of n points in the plane.

5. Design a polynomial time algorithm for finding integer b and $c > 1$, given the value of $a = b^c$. The algorithm may fail if the input a cannot be expressed in this form.

6. Show that any polynomial $p(x) \in F[x]$, where F is a field, has at most $\text{degree}\big(p(x)\big)$ roots.