

## Modeling and Simulation

## Test 2

Time: 90 min

Answer all questions

28th Oct, 2005

1. Consider the LCG  $x_{n+1} = (ax_n + c) \bmod m$ , with  $m = 10^c$ , and  $c = 3$ .  
How should we choose  $a$  to obtain a full period generator. 5 marks
2. For a LCG let  $a = 211$ . Find the potency of the generator if  $m = 15,000$ .  
5 marks
3. Briefly describe the serial test. 5 marks
4. Briefly describe the gap test. 5 marks
5. Consider the following cdf

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4}(x+1)^2 & 0 \leq x \leq 1 \\ 1 & x \geq 1 \end{cases}$$

Give an efficient algorithm to simulate this cdf. Justify your answer. Your method should not use any square root operation. 10 marks

6. Assume that you have an *expo*(1) RV generator.
  - (a) Give an algorithm to simulate a *geometric*(p) RV
  - (b) Give an algorithm to simulate a *poisson* RV
  - (c) Simulate the RV with pmf

$$P(X = i) = \frac{e^{-\mu} \frac{\mu^i}{i!}}{\sum_{i=1}^{20} e^{-\mu} \frac{\mu^i}{i!}} \quad i = 0, 1, \dots, 20$$

15 marks

7. State the Box-Muller algorithm to simulate Normal random variables. What is the average number of iterations required. 5 marks

8. Give an efficient method to simulate  $Gamma(\frac{1}{2}, 2)$  RV. Justify your answer. 10 marks
9. Let  $X \sim Beta(10, 20)$ . Give three algorithms to simulate this distribution. 15 marks