Modeling and Simulation Test 1 Time: 90 min Answer all questions

10th Sep, 2005

1. Let Ω be the set of all non-negative integers. Let S be the set of all possible subsets of Ω . Let P be a function of Ω , S given as follows

(a)

$$P(A) = \sum_{x \in A} p(1-p)^x \quad A \in \mathcal{S}$$

(b) P(A) = 1 if A is finite, and P(A) = 0 otherwise for all $A \in S$.

In each of the cases check whether the function is a probability. 2 marks

- 2. Compute P(A|B) in terms of P(A) and P(B) when $A \subseteq B$ and $B \subseteq A$. Given two independent events A and B show that A^c and B are also independent. 3 marks
- 3. Find k such that P(X = k) is maximized, when $X \sim Poisson(\lambda)$ 5 marks
- 4. Find x such that the density function f(x) is maximized for a RV $X \sim Beta(m, n)$. 5 marks
- 5. Show that $Var(X) \leq E(X-c)^2$ for any c. For what value of c equality holds. 5 marks
- 6. Let X be a RV taking non-negative integer values, such that

$$P(X > m + 1 | X > m) = P(X \ge 1) \quad m = 0, 1, \dots$$

Find the p.m.f of X.

10 marks

- 7. Let X and Y be two continuous RVs with density function f_X and f_Y . Find P(X < Y). 5 marks
- 8. Let $X_1 \sim Poisson(\lambda_1)$ and $X_2 \sim Poisson(\lambda_2)$ be independent RVs. Find the conditional p.m.f of X_1 given $X_1 + X_2$. 10 marks

- 9. It is given that the correlation coefficient $\rho_{XY} = -1$. It is also given that the variance of Y is same as that of X. Can you find the relationship between X and Y if it is given that the mean of X and mean of Y add up to 2. 10 marks
- 10. Let $X \ge m > 0$. Is the following true

$$E(\sqrt{X_2 - m_2}) \le \sqrt{\{E(X)\}^2 - m^2}$$

Give reasons.

10 marks