Modeling and Simulation

Final Exam Time: 120 min

Answer all questions

6th Dec, 2005

1. A coin is tossed n times. What is the probability of the event that the number of heads in n tosses is odd? For the coin the probability of head appearing is p. 5 marks

2. Let $X_i \sim Geom(p_i)$ be n independent RVs. What is the p.m.f. of

$$X = min(X_1, X_2, \ldots, X_n).$$

5 marks

3. Let X and Y be two independent continuous RVs with density functions f_X and f_Y respectively. What is the density function of $Z = XY$ 5 marks

4. Show that for all $a > 0$

$$P(e^{tX} \geq a) \leq \frac{M(t)}{a}$$

where $M(t)$ is the moment generating function. Assume that X is continuous RV. 10 marks

5. Let $g(t) = logM(t)$. It is given that for a Gaussian RV $g(1) = 1$ and $g(2) = 4$. Compute the density of the guassian? 5 marks

6. For a LCG, let $c = 0, m = 2^{31}$. It is given that $a = 2^{16} + 3$ or $a = 2^{16} + 5$. What value of a shall we choose. Give reasons. 5 marks

7. It is known that a RV has density 0 everywhere else except in the interval $(-a, a)$ where a is some positive constant. In the interval $(-a, a)$ the density is proportional to $e^{-(\frac{x}{a})^2}$. Give an efficient method to simulate such a RV. Justify your method. 10 marks

8. Let the hazard rate of a RV be $h(t) = 2\lambda t$. Give an efficient algorithm to simulate such a distribution. 5 marks

9. Describe the control variate technique. Consider estimating $P(X \leq a)$ by the estimator

$$I = \begin{cases} 1 & X \leq a \\ 0 & \text{otherwise} \end{cases}$$

Using X as a control variate give a new estimator. Discuss how much reduction in variance is obtained if $X \sim U(0,1)$. 10 marks

10. Briefly describe the Maximum likelihood procedure for estimating parameters. Let $X \sim U(a,b)$. From n independent observations of X find the maximum likelihood estimate of a and b. 10 marks