Modeling and Simulation Assignment 3 Due Date: 14th Nov, 2005

November 3, 2005

- 1. The moment generating function (MGF) of a RV is defined as $M(t)=E(e^{tX}).$ Find M(t) for
 - (a) $X \sim \text{Bernoulli}(p)$
 - (b) $X \sim \text{Geom}(p)$
 - (c) $X \sim N(0,1)$
 - (d) $X \sim \text{Gamma}(\alpha, \beta)$

8 marks

- 2. Let $Y = \sum_{i=1}^{n} Z_i$, where i.i.d. $Z_i \sim \text{Geom}(p)$. Find M(t) for Y. 2 marks
- 3. Consider the RV obtained by counting the number of independent tosses till r heads occur by tossing a coin with bias p. What is the p.m.f of such a random variable. Compute the MGF of such a RV. Give an efficient method for simulating such a distribution. 10 marks
- 4. Let $Y = \sum_{i=1}^{n} Z_i^2$, where $Z_i \sim N(\mu, \sigma^2)$. Find M(t) for Y. What is the distribution of Y. 5 marks
- 5. Let X and Y be continuous RVs with pdfs f_1 and f_2 respectively. Find the density function of the RV $T = \frac{X}{Y}$. 5 marks
- 6. Define the joint MGF of a *n* dimensional RV (Y_1, \ldots, Y_n) and a univariate RV X as $M(t, t_1, \ldots, t_n) = E(e^{tX + \sum_{i=1}^n t_i Y_i})$. Let Y be a *n* dimensional RV independent of a univariate RV X. Show that ¹

$$M(t, t_1, t_2, \dots, t_n) = M(0, t_1, t_2, \dots, t_n)M(t, 0, \dots, 0)$$

5 marks

7. Let X be a sample mean of n i.i.d. RVs $X_i \sim N(\mu, \sigma^2)$.

¹One can also prove the converse.

- (a) Show that X and Y are independent where $Y = (X_1 X, \dots, X_n X)$.
- (b) Let S^2 be the sample variance. Argue that S^2 is independent of X. What is the distribution of $(n-1)S^2/\sigma^2$ and X.
- (c) Compute the distribution of $T = \frac{X}{\sqrt{\frac{S^2}{n}}}$ by using question 4.

15 marks

8. For the dataset *z.mat* compute X and S^2 . We will reject the hypothesis that $X = \mu$ if

$$|X-\mu| \ge \sqrt{\frac{S^2}{n}} t_{\frac{\alpha}{2}}$$

where $P(T \ge t_{\frac{\alpha}{2}}) = \frac{\alpha}{2}$ and T is defined in the pervious question. Test whether $\mu = 0, 0.5, 1$ for $\alpha = 0.01, 0.05, 0.1$. Desribe your procedure. 10 marks