Modeling and Simulation
Assignment 2
Due Date: 5th Nov, 2005

October 19, 2005

1. The cdf of Weibull distribution with parameters a and b is given by

$$F(x) = \begin{cases} 1 - e^{-(\frac{x}{b})^a} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Give an algorithm to simulate the distribution 5 marks

2. Consider the following distribution.

$$F(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2 - x & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Give two algorithms to simulate this distribution 5 marks

3. Let X be a RV with cdf $F(x)$. Define

$$Z = X \quad a \leq X \leq b$$

What is the cdf of Z. Give an acceptance rejection based algorithm to simulate Z given that there is a program to simulate X. Give any other algorithm to simulate Z. 10 marks

4. Consider the following discrete RV

$$p(X = j) = 0.5^{j+1} + \frac{2^{j-2}}{3^j} \quad j = 1, 2, \ldots$$

Give an algorithm to simulate this distribution. 10 marks

5. The density function of a continuous RV X is given by $f(x) = a_0 + a_1 x + a_2 x^2$ for $0 \leq x \leq 1$. It is 0 everywhere else. It is also given that $f(0) = f(1) = 0$. Give an acceptance rejection algorithm to simulate X. 10 marks

6. Modify the acceptance rejection algorithm so that it applies to discrete Random variables. Use the modified algorithm to generate the following distribution

$$P(X = i) = \begin{cases} 0.1 & i = 1 \\ 0.15 & i = 2 \\ 0.25 & i = 3 \\ 0.05 & i = 4 \\ 0.45 & i = 5 \end{cases}$$

10 marks

7. Briefly describe how will you use the χ^2 test to decide whether a given stream of N numbers follow the Poisson distribution with parameter μ. Dataset *r.mat* [1] (available at the webpage) was generated from a poisson source of unknown parameter μ. You are told that μ is an integer between 1 and 15. Implement the χ^2 test for all values of μ. Report your results and from it deduce the value of μ. 25 marks

8. Briefly describe the K.S.test to decide whether a given stream of randome numbers follow the gaussian distribution with mean μ and variance σ^2. Dataset *z.mat* (available at the webpage) was generated from a gaussian distribution whose parameters are unknown. You are told that $-1 \leq \mu \leq 1$ and $1 \leq \sigma^2 \leq 5$. Implement the K.S.test. Report your results by varying μ and σ^2 in steps of 0.5. What can you say about the values of μ, σ^2. 25 marks

[1] To load this file do *loadr* in matlab