## Formal Methods in Computer Science Midterm Examination

Max Marks: 25, Weightage: 20%, Time: 2 hours

Consider the toy shown below. A marble is dropped in at either A or B and the position of the levers x,y and z cause it to come out at either C or D. Each time a marble hits a lever, it causes the lever to change direction, so that the next marble coming its way will take the opposite branch. Thus if the levers of the toy are initially as shown in the diagram, a marble dropped in at B will come out at C but if a marble is dropped in at B a second time it will come out at D.



Model the toy as an automaton which runs on inputs 'A' and 'B' (representing a marble dropped in at A and B respectively), and which accepts all sequences of inputs in which the last marble comes out at D. (4)

- 2. Prove or disprove:  $(r+s)^* = r^* + s^*$ . (2)
- 3. Let L be a regular set. Which of the following are regular? Justify your answers. (6)
  - (a)  $mid\text{-thirds}(L) = \{v \mid \exists u, w : |u| = |v| = |w| \text{ and } uvw \in L\}.$
  - (b)  $\{w \mid ww \in L\}.$
  - (c)  $\{w \in \{a, b\}^* \mid |w| \text{ is a perfect square}\}.$

4. Minimize the following DFA:



5. If L is a regular language over  $\Sigma$ , what can you say about the minimum DFA's for L and  $\Sigma^* - L$ ? (2)

(4)

6. Describe the Myhill-Nerode equivalence classes for the language (2)

 $\{w \in \{a, b\}^* \mid w \text{ contains the substring } aababa\}.$ 

7. Consider the two ways of defining when a function  $f : \mathbb{N} \to \mathbb{N}$  is regularity preserving. For a language L over an alphabet  $\Sigma$  define:

$$P_f(L) = \{ x \in \Sigma^* \mid \exists y : |y| = f(|x|) \text{ and } xy \in L \} \\ Q_f(L) = \{ x \in \Sigma^* \mid \exists y : |y| = f(|x|) \text{ and } y \in L \}.$$

The definition given in class said that f was regularity preserving iff  $P_f(L)$  is regular whenever L is regular. An alternate definition says that f is regularity preserving iff  $Q_f(L)$  is regular whenever L is regular. Argue that the two definitions are equivalent. (5)