Formal Methods in Computer Science

Final Examination

Max Marks: 40, Weightage: 40%, Time: 3 hours

1. Give a simpler regular expression for the language $(a^*)^* + (b^*a)^*$. *(2)*

2. Describe a DFA for the language $a^*b^* + b^*a^*$. What are the Myhill-Nerode equivalence classes for this language? *(4)*

3. Consider the grammar G below: *(5)*

$$S \longrightarrow aS \mid aSbS \mid \epsilon.$$

   (a) Describe five strings in $L(G)$ with different numbers of b's.

   (b) Argue that $L(G)$ is the set of all $x \in \{a,b\}^*$ such that each prefix of x has at least as many a's as b's.

4. Convert the following context-free grammar to Chomsky Normal Form: *(4)*

$$\begin{array}{lcl} S & \longrightarrow & AAA \mid B \\ A & \longrightarrow & aA \mid B \\ B & \longrightarrow & \epsilon \end{array}$$

5. Describe the language accepted by the PDA whose transitions are given below. The PDA accepts by empty stack. *(2)*

$$\begin{array}{lcl} (s,a,\perp) & \rightarrow & (s,A\perp) \\ (s,a,A) & \rightarrow & (s,AA) \\ (s,a,B) & \rightarrow & (s,\epsilon) \\ (s,b,B) & \rightarrow & (s,BB) \\ (s,b,\perp) & \rightarrow & (s,B\perp) \\ (s,b,A) & \rightarrow & (s,\epsilon) \\ (s,\epsilon,\perp) & \rightarrow & (t,\epsilon) \end{array}$$

6. Prove that it is decidable whether a given DFA accepts a finite language. Show that it is also decidable whether a given CFG accepts a finite language? (hint: use ideas from the proof of the pumping lemma). *(4)*

7. The grammar G below is an unrestricted grammar:

$$\begin{array}{lcl} S & \longrightarrow & bSb \mid AcA \\ Ab & \longrightarrow & A \mid b \\ bA & \longrightarrow & A \mid b. \end{array}$$

   The grammar works in much the same way as CFG's: begin with start symbol S, and apply the productions by replacing substring which match the left-hand side of a production by the right-hand side of the production.

The set of terminal strings derived in this way is the language generated by the grammar.

Let G' be the unrestricted grammar obtained from G by removing the production $bA \longrightarrow A$. Then one of the $L(G)$ and $L(G')$ is regular while the other is context-free but not regular: Which is which? *(6)*

8. Describe the function from a, b^* to a, b^* defined the Turing machine M below. M has as its set of states s, p, q, u, t, input alphabet a, b, left-end marker $\vdash$, and blank symbol $\flat$. s is the start state and t is the accept state. The transition relation is given as follows: *(4)*

$$
\begin{array}{lcl}
(s, \vdash) & \rightarrow & (s, \vdash, R) \\
(s, b) & \rightarrow & (s, b, R) \\
(s, \flat) & \rightarrow & (u, a, L) \\
(u, b) & \rightarrow & (u, a, L) \\
(u, \vdash) & \rightarrow & (t, \vdash, R) \\
(s, a) & \rightarrow & (p, a, R) \\
(p, a) & \rightarrow & (p, a, R) \\
(p, b) & \rightarrow & (p, b, R) \\
(p, \flat) & \rightarrow & (q, \flat, L) \\
(q, b) & \rightarrow & (q, a, L) \\
(q, a) & \rightarrow & (t, b, L)
\end{array}
$$

9. Are the following problems decidable? Justify your answer: *(4)*

   (a) Given a Turing machine M, is $L(M) = rev(L(M))$?

   (b) Given a Turing machine M, are there infinitely many Turing machines which accept the same language as M?

10. Prove that a language L is recursive iff it can be enumerated in increasing lexicographic order.

    Using this fact, prove that every infinite r.e language must have an infinite recursive subset. *(5)*

11. *This question is meant only for those who did not give a seminar during the course.*

    Give languages over a single letter alphabet which lie in each of the following classes - or explain why such languages cannot exist:

    (a) Regular languages

    (b) Deterministic Context-Free (DCFL) but not regular

    (c) CFL but not DCFL

    (d) Recursive languages but not CFL

    (e) r.e. but not recursive

    (f) Not r.e.