Formal Methods in Computer Science

Assignment 5

(Due on Monday 5 Dec 2005)

1. Prove that the following question is undecidable: Given a Turing machine M and a state q of M, does M ever enter state q on *some* input?

2. Prove that it is undecidable whether two Turing machines accept the same language.

3. Let $L, K \subseteq \Sigma^*$. Define

$$L/K = \{x \mid \exists y \in K,\ xy \in L\}$$

   (a) Show that if L is regular and K is *any* language, then L/K is regular.

   (b) Show that even if we are given a DFA for L and a Turing machine for K, we cannot always construct an automaton for L/K.

4. Show that neither the language

$$\text{TOTAL} = \{M \mid M \text{ halts on all inputs}\}$$

   nor its complement is r.e.

5. Which of the following problems are decidable and which are not? Justify your answer as usual.

   (a) Given a TM M and a string y, does M ever write the symbol # on its tape on input y?

   (b) Given a CFG G, does G generate all strings except ϵ?

   (c) Given a TM M, are there infinitely many TM's equivalent (i.e. accepting the same language) to M?

6. Is is decidable to tell whether a given Turing machine M is *minimal* or not (i.e. does there exist another Turing machine N with fewer states than M accepting the same language)?.