

## Formal Methods in Computer Science

### Assignment 4

(Due on Tuesday 22nd Nov 2005)

1. Give Pushdown Automata (PDA's) for each of the following languages. If possible, give deterministic PDA's for the languages. Specify all transitions of the PDA's.
  - (a)  $\{wcw^R \mid w \in \{a, b\}^*\}$ .
  - (b)  $\{w \in \{a, b\}^* \mid \#_a(w) = 2\#_b(w)\}$ .
2. Recall Parikh's theorem for CFL's. Show that
  - (a) Semi-linear sets are closed under complement.
  - (b) The language  $\{a, b\}^* - \{a^n b^{n^2} \mid n \geq 0\}$  is *not* a CFL.
3. Show that the following functions are computable by a Turing Machine in the sense discussed in class. Give a complete description of the moves of the TM.
  - (a)  $square : \mathbb{N} \rightarrow \mathbb{N}$  where  $square(n) = n^2$ .
  - (b) (integer division)  $div : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ , where  $div(m, n)$  is the largest integer less than or equal to  $m/n$  if  $n \geq 0$  and 0 otherwise.
  - (c)  $exp : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$  where  $exp(m, n) = m^n$ .
4. Show that the following languages over  $\Sigma = \{a, b\}$  are recursive. Describe your TM's fully.
  - $L = \{ww \mid w \in \{a, b\}^*\}$ .
  - $L = \{a^{2^n} \mid n \geq 0\}$ .