Formal Methods in Computer Science

Assignment 3

(Due on Thu 27th Oct 2005)

1. Describe the languages accepted by the following grammars:

   (a)
   $$\begin{aligned} S &\rightarrow AA \mid 0 \\ A &\rightarrow SS \mid 1 \end{aligned}$$

   (b)
   $$S \rightarrow bS \mid Sa \mid aSb \mid \epsilon$$

2. A *regular* grammar (or a *strongly right-linear* grammar) is a grammar in which all rules are of the form $X \rightarrow aY$ or $X \rightarrow \epsilon$ where a is a termninal and X and Y are non-terminals. Prove that the class of languages definable by regular grammars is precisely the class of regular languages.

3. Give context-free grammars for the following languages

   (a) $L_1 = \{a^i b^j c^k \mid i = j\}$

   (b) $L_2 = a^*b^*c^* - \{a^n b^n c^n \mid n \geq 0\}$.

4. Give an equivalent grammar in Chomsky Normal Form for the following CFG:
   $$\begin{aligned} S &\rightarrow aSbb \mid T, \\ T &\rightarrow bTaa \mid S \mid \epsilon. \end{aligned}$$

5. Give a context-free grammar for the following language. Prove that your grammar is correct: "Equal a's and b's" – i.e. $\{x \in \{a,b\}^* \mid \#_a(x) = \#_b(x)\}$. (Hint: give a grammar similar to the one for balanced parenthesis).

6. Prove that the intersection of a CFL and a regular language is a CFL.

7. Prove or disprove: CFL's are closed under the asterate (*) operator.

8. Show that $L = \{a^i b^j \mid i = j^2\}$ is not a CFL.

9. Let $\Sigma = \{a, b, c\}$. Exactly one of the statements below is true. Give a proof for the correct one and counter examples for the other three:

   (a) For any $L \subseteq \Sigma^*$, if L is regular then so is $\{xx \mid x \in L\}$.

   (b) For any $L \subseteq \Sigma^*$, if L is regular then so is $\{x \mid xx \in L\}$.

   (c) For any $L \subseteq \Sigma^*$, if L is context-free, then so is $\{xx \mid x \in L\}$.

   (d) For any $L \subseteq \Sigma^*$, if L is context-free then so is $\{x \mid xx \in L\}$.