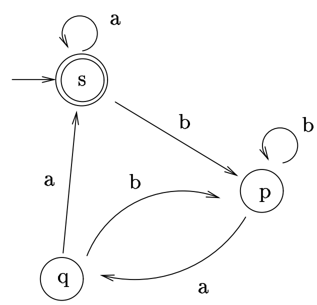
Formal Methods in Computer Science

Assignment 3

(Due on Tuesday 4th Oct 2005)

1. Use the construction done in class to construct a regular expression corresponding to the language accepted by the DFA below (i.e. the expression corresponding to $L_{ss}^{\{s,p,q\}}$).



2. Consider the language BP of balanced parenthesis. Thus BP contains strings of the form "((())())" but not "(()(". Give an argument using the pumping lemma to show that BP is not regular.

3. Use the ultimate periodicity property of regular languages to show that the language $\{a^{2^n} \mid n \geq 0\}$ is not regular.

4. For a set of natural numbers A, define *binary*(A) to be the set of binary representations of numbers in A. Similarly define *unary*(A) to be the set of "unary" representations of numbers in A: *unary*$(A) = \{1^n \mid n \in A\}$. Thus for $A = \{2, 3, 6\}$, *binary*$(A) = \{10, 11, 110\}$ and *unary*$(A) = \{11, 111, 111111\}$.

   Consider the two propositions below:

   (a) For all A, if *binary*(A) is regular then so is *unary*(A).

   (b) For all A, if *unary*(A) is regular then so is *binary*(A).

   One of the statements above is true and the other is false. Which is true and which is false?

5. Give a language $L \subseteq \{a, b\}^*$ such that neither L nor $\{a, b\}^* - L$ contains an infinite regular set.

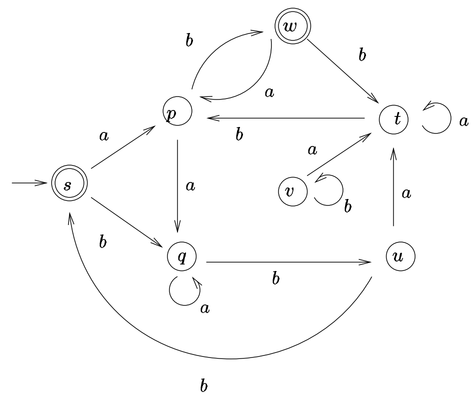
6. Let L be an arbitrary subset of $\{a\}^*$. Prove that L^* is regular by showing that *lengths*(L^*) is ultimately periodic with period the length of the smallest non-null string in L.

7. Describe the equivalence classes of the Myhill-Nerode relation $\equiv_L$ for the languages:

   (a) L is set of strings over $\{a, b\}$ in which the 3rd last letter is a b.

   (b) $L = \{x \in \{a, b\}^* \mid \#_a(x) = \#_b(x)\}$.

8. Minimize the following DFA using the collapsing algorithm done in class. Point out the states in the original DFA which correspond to the states of the minimized DFA.



9. Recall that a function $f : \mathbb{N} \to \mathbb{N}$ is said to be *ultimately periodic* if there exists $p \geq 1, n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $f(n) = f(n+p)$. Further, f is said to be ultimately periodic mod m if the function $n \mapsto (f(n) \mod m)$ is ultimately periodic. Show that the functions $n \mapsto n^2$ and $n \mapsto 2^n$ are ultimately periodic mod m, for each $m \geq 1$.

10. Give an explicit construction to show that the language

$$\{x \in \{a, b\}^* \mid \exists y : |y| = |x|^2 \text{ and } xy \in L\}$$

is regular for any regular language L over $\{a, b\}$.