Formal Methods in Computer Science

Assignment 1

(Due on Thu, 25 Aug 2005)

1. Call a language L *reflexive* if $\epsilon \in L$ and *transitive* if $L \cdot L \subseteq L$. Show that L^* is the smallest reflexive and transitive language containing L.

2. Give DFA's for each of the following languages over $\Sigma = \{a, b\}$:

   (a) All strings which do *not* contain the substring $babb$. (By a *substring* we mean a contiguous sequence of letters: thus u is a *substring* of w if there exist strings v_1 and v_2 such that $w = v_1 u v_2$.)

   (b) Between every pair of b's there is at least one a.

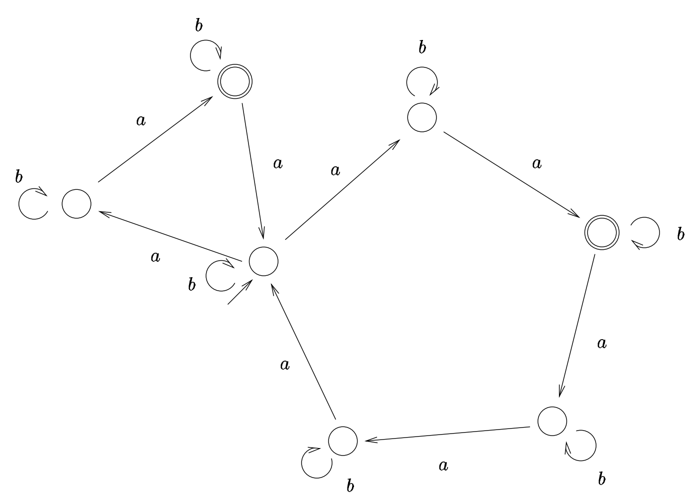
   (c) Strings which contain both aa and bb as substrings.

   (d) all strings which have at least three positions where 3 consecutive b's occur. For example $abbbbb$ and $bbbbabbba$ should be accepted, but not $bbba$.

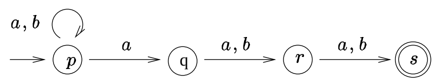
3. Show that the set of strings in $\{0, 1, 2\}^*$ which are base 3 representations of even numbers, is regular.

4. Give DFA's for the languages $L_1 = \{x \in \{a, b\}^* \mid |x| = 0 \mod 2\}$ and $L_2 = \{x \in \{a, b\}^* \mid |x| = 0 \mod 3\}$. Also give the product DFA which accepts $L_1 \cap L_2$.

5. Describe the language accepted by the NFA below:



6. Consider the NFA below:



(a) Use the subset construction to obtain an equivalent DFA for the NFA below. Label each state of the DFA with the subset of states of the NFA that it corresponds to.

(b) Give an 8 state DFA which accepts the same language.

7. Recall that for the transition function δ of a DFA we had defined the "string" version of δ, called δ^*, inductively on the length of strings, as follows:

$$\begin{aligned} \delta^*(q, \epsilon) &= q \\ \delta^*(q, wa) &= \delta(\delta^*(q, w), a). \end{aligned}$$

For the transition relation $\longrightarrow$ of an NFA, we had defined the relation $\longrightarrow^*$ as follows:

$$\begin{array}{lll} p \overset{\epsilon}{\longrightarrow}{}^* p & & \\ p \overset{wa}{\longrightarrow}{}^* q & \text{iff} & \exists r \text{ such that } p \overset{w}{\longrightarrow}{}^* r \text{ and } r \overset{a}{\longrightarrow} q. \end{array}$$

Finally, in the definition of the subset automaton for a given NFA we defined the transition relation δ of the subset automaton as $\delta(X, a) = \{q \mid \exists p \in X : p \overset{a}{\longrightarrow} q\}$.

Prove formally that the transition function δ of the subset automaton satisfies:

$$\delta^*(X, w) = \{q \mid \exists p \in X : p \overset{w}{\longrightarrow}{}^* q\}.$$

8. Answer the following questions about NFA's:

(a) Argue that if an NFA with k states accepts any string at all, then it accepts one of size $k - 1$ or less.

(b) Give an NFA over a single letter alphabet that rejects some string, but the length of the shortest rejected string is strictly more than the number of states in the NFA.

(c) Give a construction for arbitrary large NFA's which shows that the length of the shortest rejected string can be exponential in the number of states of the NFA.