Game Theory in Social Networks Analysis

Abstract

This paper is aimed towards providing an introduction to Social Networks and tries to provide how game theory can be an useful tool towards analysis of social networks. We are trying to make this paper useful for both game theory and social networks community. This paper wherever necessary will provide necessary details of game theory and social networks. The course of the paper will be

1. Introduction to Social Networks
2. Types of Social Networks
3. Representation of Social Networks
4. Analysis on Social Networks
5. Representation of Social Networks as Games
6. Valuation of nodes of Social Networks
    a. Cooperative Version
    b. Non-Cooperative Version
7. Analysing network formation in Social networks
    a. Matching problem
8. Economic Applications of Game Theoretic Analysis of Social Networks
    a. Competing for Customers in Social Network

Introduction to Social Networks

A social network is a social structure made of nodes which are generally individuals or organizations. The maximum size of social networks tends to be around 150 people (Dunbar's number) and the average size around 124 (Hill and Dunbar, 2002).

Social network analysis is a method of inquiry that focuses on relationships between subjects such as individuals, organisations or nation states. However, it has also been applied to analysis relationships between objects as diverse as the Internet, scientific papers, organisms, and molecules.

Social network analysis [SNA] is the mapping and measuring of relationships and flows between people, groups, organizations, animals, computers or other information/knowledge processing entities. The nodes in the network are the people and groups while the links show relationships or flows between the nodes. SNA provides both a visual and a mathematical analysis of human relationships. Management consultants use this methodology with their business clients and call it Organizational Network Analysis.

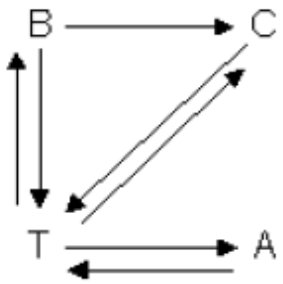
A social network is a set of actors (or points, or nodes, or agents) that may have relationships (or edges, or ties) with one another. On one hand, there really isn't anything about social network data that is all that unusual. The major difference between conventional and network data is that conventional data focuses on actors and attributes; network data focus on actors and relations. “Conventional" sociological data consists of a rectangular array of measurements. The rows of the array are the cases, or subjects, or observations. The columns consist of scores (quantitative or qualitative) on attributes, or variables, or measures. Each cell of the array then describes the score of some actor on some attribute. In some cases, there may be a third dimension to these arrays, representing panels of observations or multiple groups.

Representation of Social Networks

Representing Networks with Graphs

Mathematicians know the kind of graphic displays by the names of "directed graphs" “signed graphs" or simply "graphs."

Let's suppose that we are interested in summarizing who nominates whom as being a "friend" in a group of four people (Bob, Carol, Ted, and Alice). We would begin by representing each actor as a "node" with a label (sometimes notes are represented by labels in circles or boxes).



Kinds of Graphs

Levels of Measurement: Binary, Signed, and Valued Graphs

But, we could have asked the question a second way: "for each person on this list, indicate whether you like,dislike, or don't care." We might assign a + to indicate "liking," zero to indicate "don't care" and - to indicate dislike. This kind of data is called "signed" data.

Directed or "Bonded" Ties in the Graph

In our example, we asked each member of the group to choose which others in the group they regarded as close friends. Each person (ego) then is being asked about ties or relations that they themselves direct toward others (alters). Each alter does not necessarily feel the same way about each tie as ego does: Bob may regard himself as a good friend to Alice, but Alice does not necessarily regard Bob as a good friend. It is very useful to describe many social structures as being composed of "directed" ties (which can be binary, signed, ordered, or valued).

"Directed" graphs use the convention of connecting nodes or actors with arrows that have arrowheads, indicating who is directing the tie toward whom. This is what we used in the graphs above, where individuals (egos) were directing choices toward others (alters). "Co-occurrence" or "co-presence" or "bonded-tie" graphs use the convention of connecting the pair of actors involved in the relation with a simple line segment (no arrowhead).

Simplex or Multiplex Relations in the Graph

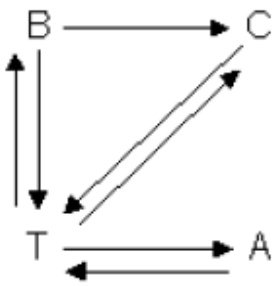
A graph that represents a single kind of relation is called a simplex graph.

Bob identifies Ted as kin; Ted identifies Bob; and Ted and Alice identify one another (the full story here might be that Bob and Ted are brothers, and Ted and Alice are spouses). We could add this information to our graph, using a different color or different line style to represent the second type of relation ("is kin of...").

A graph (sometimes called a sociogram) is composed of nodes (or actors or points) connected by edges (or relations or ties). A graph may represent a single type of relations among the actors (simplex), or more than one kind of relation (multiplex). Each tie or relation may be directed (i.e. originates with a source actor and reaches a target actor), or it may be a tie that represents co-occurrence, co-presence, or a bonded-tie between the pair of actors. Directed ties are represented with arrows, bonded-tie relations are represented with line segments. Directed ties may be reciprocated (A chooses B and B chooses A); such ties can be represented with a double-headed arrow. The strength of ties among actors in a graph may be nominal or binary (represents presence or absence of a tie); signed (represents a negative tie, a positive tie, or no tie); ordinal (represents whether the tie is the strongest, next strongest, etc.); or valued (measured on an interval or ratio level).

Using Matrices to Represent Social Relations

The "Adjacency" Matrix



	B	C	T	A
B	---	1	1	0
C	0	---	1	0
T	1	1	---	1
A	0	0	1	---

Sometimes the value of the main diagonal is meaningless, and it is ignored (and left blank). Sometimes, however, the main diagonal can be very important, and can take on meaningful values. This is particularly true when the rows and columns of our matrix are "super-nodes" or "blocks."

	Bob	Ted	Carol	Alice
Bob	---	1	1	0
Ted	1	---	1	1
Carol	0	1	---	0
Alice	0	1	0	---

This kind of grouping of cells is often done in network analysis to understand how some sets of actors are "embedded" in social roles or in larger entities

There are various operations that can be done on social network data which is represented as matrix. We are presenting few operation and its intuition of what kind of information you can infer from the operations.

Operation on Matrix Social Network Data

Inverse of Matrix is sort of like looking at black lettering on white paper versus white lettering on black paper:

Matrix Operations : Transpose, Inverse, Addition, Subtraction, Multiplication, Correlation, Regression, Boolean Multiplication

If I had a symmetric matrix that represented the tie "exchanges money" and another that represented the relation "exchanges goods" I could add the two matrices to indicate the intensity of the exchange relationship.

Individual actors' positions in networks are also usefully described by the numbers and lengths of pathways that they have to other actors. Actors who have many pathways to other actors may be more influential with regard to them. Actors who have short pathways to more other actors may me more influential or central figures. So, the number and lengths of pathways in a network are very important to understanding both individual's constraints and opportunities, and for understanding the behavior and potentials of the network as a whole.

One of the most common and important approaches to indexing the distances between actors is the geodesic. The geodesic is useful for describing the minimum distance between actors. The geodesic distances between pairs of actors are the most commonly used measure of closeness. The average geodesic distance for an actor to all others, the variation in these distances, and the number of geodesic distances to other actors may all describe important similarities and differences between actors in how and how closely they are connected to their entire population.

Analysis of Social Network Analysis

Quantities in Social Network Analysis

Betweenness
Degree an individual lies between other individuals in the network; the extent to which a node is directly connected only to those other nodes that are not directly connected to each other; an intermediary; liaisons; bridges. Therefore, it's the number of people who a person is connected to indirectly through their direct links.

Centrality Closeness
The degree an individual is near all other individuals in a network (directly or indirectly). It reflects the ability to access information through the "grapevine" of network members. Thus, closeness is the inverse of the sum of the shortest distances between each individual and every other person in the network.

Centrality Degree
The count of the number of ties to other actors in the network.

Centrality Eigenvector
Eigenvector centrality is a measure of the importance of a node in a network. It assigns relative scores to all nodes in the network based on the principle that connections to nodes having a high score contribute more to the score of the node in question.

Centralization
The difference between the n of links for each node divided by maximum possible sum of differences. A centralized network will have much of its links dispersed around one or a few nodes, while a decentralized network is one in which there is little variation between the n of links each node possesses

Clustering Coefficient
The clustering coefficient is a measure of the likelihood that two associates of a node are associates themselves. A higher clustering coefficient indicates a greater 'cliquishness'.

Cohesion
Refers to the degree to which actors are connected directly to each other by cohesive bonds. Groups are identified as ‘cliques’ if every actor is directly tied to every other actor, ‘social circles’ if there is less stringency of direct contact, which is imprecise, or as structurally cohesive blocks if precision is wanted.

Density
Individual-level density is the degree a respondent's ties know one another/ proportion of ties among an individual's nominees. Network or global-level density is the proportion of ties in a network relative to the total number possible (sparse versus dense networks).

Path Length
The distances between pairs of nodes in the network. Average path-length is the average of these distances between all pairs of nodes.

Radiality
Degree an individual’s network reaches out into the network and provides novel information and influence

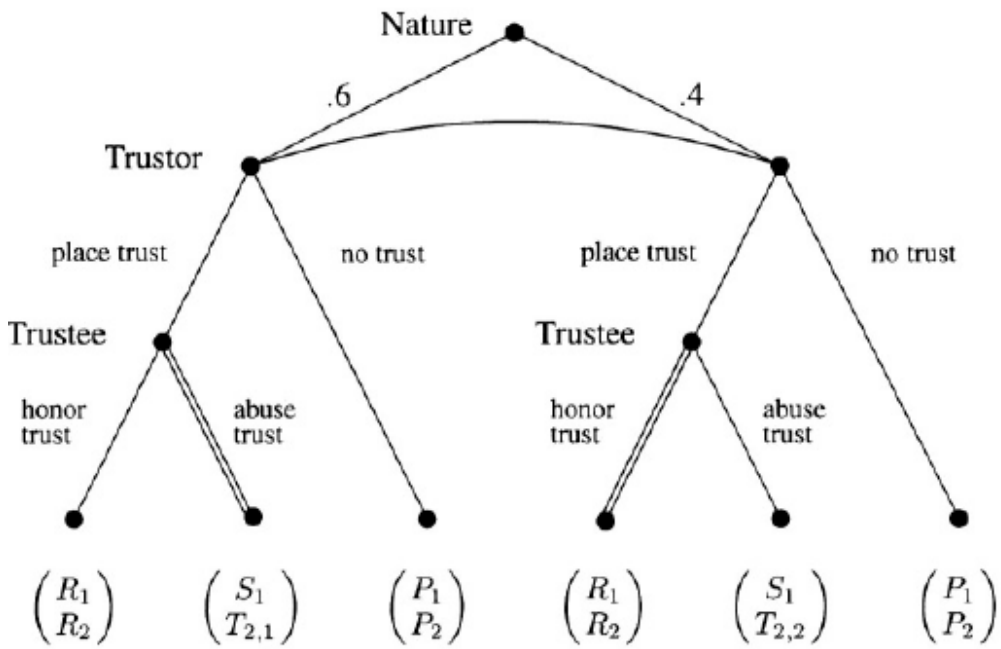
Reach
The degree any member of a network can reach other members of the network. See also reach.

Structural Equivalence
Refers to the extent to which actors have a common set of linkages to other actors in the system. The actors don’t need to have any ties to each other to be structurally equivalent.

Structural Hole
Static holes that can be strategically filled by connecting one or more links to link together other points. Linked to ideas of social capital: if you link to two people who are not linked you can control their communication.

There are also other measures such as Constraint, Contagion, Integration etc.

Building Trust in Social Networks – A Game Theoretic Representation



$R_i > P_i$ (i=1,2), $P_1 > S_1$, $T_{2,1} > R_2$ and $T_{2,2} < R_2$

Repeated games can be analyzed as social networks. One type of model that has found a large number of successors is similar to computer tournament in which actors are randomly matched together to play certain games. Actors in these games have prescribed strategies such as always abusing trust or playing Tit-for-Tat, but there is no network structure that facilitates information transmission about past periods among actors. Only one global property of the network, namely, the minimal outdegree in the network, affected the trigger strategy equilibria. There is a growing literature on models in which actors are assumed to be placed on a grid ("cellular automata"), which can be interpreted as a social network, and actors play with neighbors on the grid. However, in these models all actors have the same number of neighbors and equal probabilities of playing with each of these neighbors. There are some analyses in which the number of neighbors

of an actor is varied. This allows predictions about differences between networks but not within networks, because all positions in the network are equivalent. In most real-world networks there are, of course, individual differences between actors. Some actors have more contacts than others and these contacts may be more intense

Learning becomes an issue if incomplete information is introduced. Analyses of the finitely repeated Trust Game with incomplete information yield some appealing results with respect to control and learning effects The problem with the finitely repeated ordinary Trust Game is the following. If the trustee has an incentive to abuse trust, this implies that the trustee will certainly abuse trust in the last period. Therefore, the trustor will not place trust in the last period. Consequently, the trustee will abuse trust in the last period but one, because in the last period he will receive anyway. This implies that the trustor also cannot place trust in the last period but one.

This argument continues up to the first period, which means that the trustor can never place trust. The situation changes if there is even a slight probability that the trustee does not have an incentive to abuse trust. Then, the trustor might want to test whether the trustee has an incentive to abuse trust. Therefore, “some” trust is possible if there are “enough” periods to be played in the future. An equilibrium exists that consists of three phases. In the first phase, all types of trustees honor trust and, consequently, the trustor places trust. If the end of the game is approached (the exact timing depends on the parameters of the game), the bad trustee starts to randomize between abusing and honoring trust. Of course, the good trustee continues to honor trust throughout the game. As long as the trustee continues to honor trust in this randomization period, the trustor is more and more convinced that she is playing with a good trustee and, therefore, continues to trust although the end of the game comes closer and closer. As soon as the trustee abuses trust for the first time, he reveals himself as being a bad trustee and the trustor will never place trust again.

Recently, games resembling the Trust Game have been analyzed. In these games, one actor cannot observe the type of another actor. Consider a buyer-seller relationship as a starting point of their game. The seller has the possibility of selling high-quality or low-quality products. The seller has to make that decision in advance because he has to make an extra investment, for example in production technology, committing himself to one of the two strategies. He cannot change that decision later in the game. The buyers cannot directly observe the choice of the seller. The probability that the seller will make the investment depends on the time he expects to be in the market with that product and the extent to which buyers are able to evaluate whether he is selling high-quality or low-quality products. Interpreting the results in terms of social networks, sellers have a larger incentive to sell high-quality products to buyers who obtain more information about the behavior of this seller from other buyers.

Let us limit the analysis of learning effects to modeling how fast trustors can obtain information, assuming that trustors who obtain more information learn faster. Consequently, trustors who obtain more positive information can place more trust in the trustee than other trustors. Developing and analyzing a model that combines social networks with a game-theoretic model including incomplete information may well be part of future research efforts. A first model in this direction consists of two trustors who play a finitely repeated Trust Game with one trustor and can inform each other between periods about the behavior of the trustee.

More information can be found on Trust Games on the book "Social Networks and Game theory" by Buskens found in the reference section.

Game Theoretic Analysis of Social Networks

As mentioned in the paper earlier we can logically divide social network analysis into two sections. Valuation/evaluation of a node in a social network and analyzing how the network could be formed with these evaluated nodes. Till this section of the paper we analyses how to value the node. From now on wards we will look of network formation based on the evaluated nodes. This is also generally referred as matching problem in the social network literature.

Economic Appplications of Game Theoretic Analysis of Social Networks

The payoff to an individual from an economic or social activity depends on the network of connections among individuals. Predictions can be made regarding the likelihood that the stochastic process will lead to any given network at some time, where the stochastic process selects from among the statically stable networks and cycles. Networks also play fundamental roles in the payoffs earned from bargaining with an organization and in the exchange of goods and services. Networks through which financial help or insurance is exchanged in developing countries. Even standard matching problems (e.g., the marriage and college) are special situations where network relationships are important. This deterministic dynamic process may end at stable networks or in some cases may cycle. To this basic deterministic dynamic we add random perturbations and examine the distribution over networks as the level of random perturbations goes to 0. This stochastic dynamic process refines the prediction of the deterministic process and provides a robustness check to see which networks predicted by the deterministic dynamic are most stable in the face of small perturbations.

To be more specific, networks are modeled as graphs, where nodes or vertices represent individuals and links or edges represent connections between the individuals. Links are non-directed and thus reciprocal. A link

between two individuals can be formed only if both individuals agree to add the link, while a single individual can sever an existing link. Each individual receives a payoff or net benefit based on the network configuration that is in place. This payoff can be interpreted as the utility or production that an individual obtains from the social interaction that occurs through the network.

The primary tool that we introduce to analyze dynamic network formation is the concept of a sequence of networks that emerge when individuals form or sever links based on the improvement the resulting network offers relative to the current network. Such a sequence, called an ''improving path,'' has the properties that (i) each network in the sequence differs from the previous network by the addition or deletion of a single link, and (ii)the addition or deletion of the link benefits the individual(s) whose consent is necessary for the change.

Myopic behavior is natural in the context of large networks where players may have limited information about the incentives of others, and generally provides a useful starting point for the study of the evolution of networks.

The improving paths emanating from any starting network must lead to either a pairwise stable network (where no two players want to form a link, and no individual player wants to sever a link) or a cycle (where a number of networks are repeatedly visited). We show that there always exists either a pairwise stable network or a cycle from which there is no exit. We give a simple trading network example to show that it is possible for cycles to exist while pairwise stable networks fail to exist.

The process naturally gravitates to pairwise stable networks and cycles, but periodically is bumped away by the random errors or mutations. The intuition for which networks are visited most often comes from the idea of resistance. In the network context, resistance keeps track of how many errors or mutations are needed to get from some given network to an improving path leading to another network. Very roughly, networks that are harder to get away from and easier to get back to, in terms of resistance, are favored by the stochastic process (although this favoritism depends on the full configuration of resistance among different networks).

Aumann and Myerson were the first to take an explicit look at network formation in a strategic context where individuals had discretion over their connections; these connections defined a communication structure that was applied to a cooperative game. Aumann and Myerson model was later extended to a one-stage model of link formation and payoff division. However, the analysis in those papers is devoted to issues in cooperative game theory such as the characterization of value allocations.

Directed networks allow one individual to connect to another without the consent of the second individual, and thus applications are to settings such

as advertising, the sending of mail, and such. These are fundamentally different applications from the non-directed networks that we consider here where both individuals need to consent to form a relationship, which applies to social relationships such as friendship and marriage, as well as trading relationships, insurance networks, job contacts, bargaining networks, etc. Second, the directed networks end up having different incentive properties as individuals can unilaterally form new links, whereas here we need to consider the incentives of two individuals in forming a link. Also a repeated game has been examined, and focused on learning as a way to identify equilibria, which is quite different from the stochastic dynamic we examine here.

Trading Example (Nonexistence of a Pairwise Stable Network) Players benefit from trading with other players with whom they are linked, and trade can only flow along links. Players begin by forming a network. Subsequently, they receive random endowments and trade along chains of the network. Trade flows without friction along any chain and each connected component trades to a Walrasian equilibrium. The expected utility for a player of being in a given network is calculated by expecting over the Walrasian equilibria that result in the player's connected component as a function of realized endowments.

Let us take an example where, there are two goods. All players have identical utility functions for the two goods which are symmetric Cobb–Douglas, $U(x, y)=xy$. Each player has a random endowment, which is independently and identically distributed. A player's endowment is either (1,0) or (0, 1), each with probability 1/2, realized after the network is in place. For a given network, Walrasian equilibria occur on each connected component, regardless of the configuration of links. For instance, three players in a line have the same trades as three players in a circle (triangle), but with a lower total cost of links. Let the cost of a link be equal to 5/96 (to each player in the link).

Social Networks and Matching Problems

For the marriage problem, the set of players N is divided into a set of men, $M=\{m1, ..., mj\}$, and a set of women, $W=\{w1, ..., wk\}$. A network, g, is feasible if each woman is linked to at most one man, and each man is linked to at most one woman.

Hospital-Intern and College Admissions Problems

For the hospital-intern (or college admissions) problem, the set of players N is divided into a set of hospitals, $H=\{h1 , ..., hj\}$, and a set of interns, $I=\{i1, ..., ik\}$. A network, g, is feasible if each intern is linked to at most one hospital, and each hospital, h, is linked to at most qh interns, where $q_h > 0$ is the quota for the hospital; thus each hospital has a fixed number of slots.

Existence of a Cycle in a Marriage Problem.

Consider a marriage problem with two men and two women, where preferences are as
follows:
m1: w1, w2
m2: w2, w1
w1: m2, m1
w2: m1, m2 .
There exists a cycle under the definition of simultaneous improving path:
{m1w1} to {m2w1} to {m2w2} to {m1w2} to {m1w1}.

The existence of a marriage cycle was first demonstrated by Knuth and is also discussed in Roth and Vande Vate. While there are cycles in the marriage problem there are no closed cycles. That follows from the main result in Roth and Vande Vate, which in the network language says that from each network there is a simultaneous improving path leading to some core stable network. This implies (from Lemma 2 here) that all stochastically stable networks will be core table networks. So a question arises as to which core stable networks are stochastically stable. It is known (since Gale and Shapley) that the set of core stable matchings has nice properties.

Economic Applications of Game Theoretic Analysis of Social Networks

Competing for Customers in a Social Network

There are many situations in which a customer's proclivity to buy the product of any firm depends not only on the classical attributes of the product such as its price and quality, but also on who else is buying the same product. We model these situations as games in which firms compete for customers located in a "social network". Nash Equilibrium (NE) in pure strategies exist in general.

This relation between the connectivity of a customer and the money firms spend on him, is transparent when externalities are dominant.

Let us take the following setting in linear environment.

A, I - Set of firms and customer

m_{α}^{i} - Money spent by first α to customer i, $\alpha \in A$ & i $\in I$

R_{+}^{I} - Strategy set for firm $\alpha \in$ A

R_{+}^{IxA} – Strategy Profile of players

The nash equilibrium for such a game can be defined as

$$\prod^{\alpha}(m) \geq \prod^{\alpha}(\overline{m}^{\alpha}, m^{-\alpha}) \forall \overline{m}^{\alpha} \in R_{+}^{I}$$

Such a nash equilibrium will exist when

- The cost function is continuous, convex and strictly increasing
- The benefit function is continuous, convcave and increasing
- The externality function is continuous
- Customer has two distinct competing firms and customer responds to marketing of both the firms, and whose values on the customer strictly increases

Similarly in quasi-linear model, there exists a unique nash equilibrium satisfying the first order conditions that maximizes payoff as follows

$$\begin{aligned} v_i^{\alpha}\lambda_i^{\alpha}(r_i^{\alpha}, \overline{m}_i) &= c_i^{\alpha} \text{ if } m_i^{\alpha} > 0 \\ v_i^{\alpha}\lambda_i^{\alpha}(r_i^{\alpha}, \overline{m}_i) &\leq c_i^{\alpha} \text{ if } m_i^{\alpha} = 0 \\ v_i^{\alpha}\lambda_i^{\alpha}(s_i^{\alpha}, \overline{\eta}_i) &= c_i^{\alpha} \text{ if } \eta_i^{\alpha} > 0 \\ v_i^{\alpha}\lambda_i^{\alpha}(s_i^{\alpha}, \overline{\eta}_i) &\leq c_i^{\alpha} \text{ if } \eta_i^{\alpha} = 0 \end{aligned}$$

This unique nash equilibrium can be characterized as

$$m_i^{\alpha} = \left(\frac{k_i - 1}{\sum_{\beta=1}^{k_i} \kappa_i^{\beta}}\right)\left(1 - \frac{(k_i - 1)\kappa_i^{\alpha}}{\sum_{\beta=1}^{k_i} \kappa_i^{\beta}}\right)$$

Where firms 1…k will spend money on the customer i as represented above in the nash equilibrium.

This paper further discusses about the impact of competition and anonymous valuation on the multiple nash equilibrium and also analyses a markov chain perspective when externalities become dominant.

Conclusion

Game theory can be applied in the various perspectives of social networks. Game theory and mechanism design can be used as the tool to analyse the value of node in the social network. These tools can also be used to consider for the network formation in the social network. Value Theory and social theory is the two areas to look at for understanding how to value a node in the network. There has been work in the area of applying cooperative game theory (such as shapley value) in the valuation of a node in the social network. Also Orkut is using a mechanism design technique to value players in orkut.

Mechanism Design and Game theory can be used to analyzing and building exciting application on top of social networks. This could be chosen as a possible area for research. Data mining and game theory can be applied together to solve many exciting problems of social network.

Building application with game theory and mechanism design on social network is a promising area. Hence Game theory and mechanism design will be used a tool to analyse social networks.

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