Game Theory in Social Networks

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Agenda

- Definition of Social Networks
- Introduction to Social Networks
  - Analysis of Social Networks
  - Representation of Social Networks
- Social Network Analysis
  - Node
    - Cooperative Version using Shapley
  - Network Formation
    - Matching Networks
      - Social Networks and Matching Problem
- Application of Social Networks
  - Competing for Customers in a Social Network

Definition of Social Networks

- A social network is a map of the relationships between individuals, indicating the ways in which they are connected through various social familiarities ranging from casual acquaintance to close familial bonds. The term was first coined in 1954 by J. A. Barnes (in: Class and Committees in a Norwegian Island Parish, "Human Relations").

source : http://en.wikipedia.org/wiki/Social_networks

Few Examples

- List of Social Networking websites can be found at http://en.wikipedia.org/wiki/List_of_social_networking_websites .
- Business - LinkedIn
- Friends ‘n’ Family - Gazzag, orkut, Yahoo 360
- Knowledge – wikipedia

Introduction to Social Networks

- Social Network Analysis
  - mapping and measuring of entities/relationships and flows between entities
  - visual and a mathematical analysis of relationships
  - Measurements, operations
- Data
  - Matrix. The major difference between conventional and network data is that conventional data focuses on actors and attributes; network data focus on actors and relations
- Representation
  - Graphs.

Representation of Social Networks

- Graphs
  - Levels of Measurement: Binary, Signed, and Valued Graphs (eg. Procurement networks)
  - Directed or "Bonded" Ties in the Graph
  - Simplex or Multiplex Relations in the Graph
  - Trees – Iterated Games
- Matrix
  - Adjacency
  - Utility Matrix

Analysis of Social Networks

- Measurements
  - Betweenness - the number of people who a person is connected to indirectly through their direct links.
  - Centrality Closeness - the inverse of the sum of the shortest distances between each individual and every other person in the network.
  - Centrality Degree - The count of the number of ties to other actors in the network.
  - Centralization - A centralized network will have much of its links dispersed around one or a few nodes, while a decentralized network is one in which there is little variation between the number of links each node possesses
  - Reach - The degree any member of a network can reach other members of the network.

Operations on Social Network Data

- Transpose – correlation between an adjacency matrix and the transpose of that matrix is a measure of the degree of reciprocity of ties
- Inverse - Inverse of Matrix is sort of like looking at black lettering on white paper versus white lettering on black paper
- Addition and Subtraction – Let a symmetric matrix that represented the tie "exchanges money" and another that represented the relation "exchanges goods" addition of the two matrices indicate the intensity of the exchange relationship.
- Multiplication and Boolean matrix multiplication - there is a path from each actor to each actor

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Value of a Node

- Computation of values of node using shapley value.

$$\varphi_i(v) = \sum_{c \subseteq N \setminus \{i\}} \frac{|c|!(|N|-|c|-1)!}{N!}(v(c \cup \{i\}) - v(c))$$

- Challenge
  - Measure of coalitions
- Reference
  - Social and Economic Networks in Cooperative Game Theory

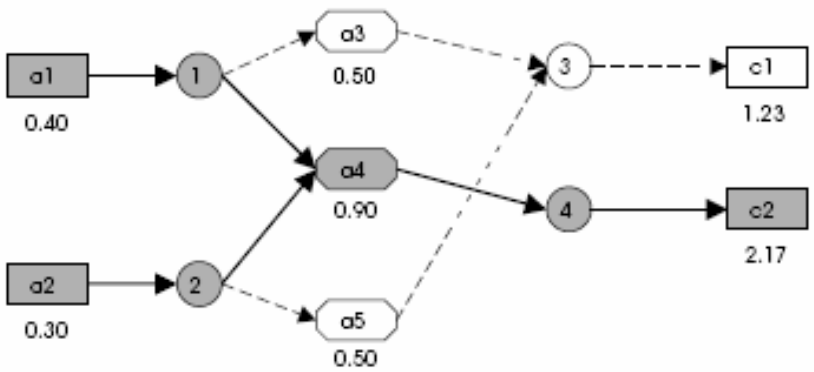
Network Formation – Marriage Formation

- Consider a stable mechanism where each agent announces his/her own preferences and a stable matching is selected for the announced preference profile. The mechanism implements the stable rule that selects all of the stable matchings in a strong equilibrium. In other words, when selected any stable matching to an announced preference profile, in fact the matching is stable for the true preference profile in environments where agents' behaviors are described by a strong Nash equilibrium

Network Formation - Room Allocation Problem

- Many universities in the U.S. employ some variant of a mechanism called the random serial dictatorship with squatting rights (RSD) to allocate dormitory rooms. Each existing tenant can either keep her house or enter the applicant pool. Each applicant is randomly given a priority and each is assigned, in priority order, her top choice among the houses that remain. This mechanism is strategy-proof for each applicant, reporting her true preferences is a dominant strategy. An existing tenant who enters the lottery may end up with a house less preferred than her current house and therefore this mechanism is not individually rational. Consequently not every existing tenant will join the applicant pool and potential gains from trade is lost. Therefore RSD is not Pareto efficient

Network Formation – Procurement Formation

- Setting
  - Combinatorial auctions, incentive compatible, one-shot, supply chain surplus, BNE for special class of networks
- Task Dependency Network
  - 

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Applications

- There are many situations in which a customer's proclivity to buy the product of any firm depends not only on the classical attributes of the product such as its price and quality, but also on who else is buying the same product. We model these situations as games in which firms compete for customers located in a "social network". Nash Equilibrium (NE) in pure strategies exist in general.
- Classical Examples
  - Cell-phone connections
  - Competition for advertisement on the web

Results

- What is Nash Equillibrium

$$A, I \text{ - Set of firms and customer}$$

$$m_{\alpha}^{i} \text{ - Money spent by first } \alpha \text{ to customer i}, \alpha \in A \text{ \& i} \in I$$

$$R_{+}^{I} \text{ - Strategy set for firm } \alpha \in \text{A}$$

$$R_{+}^{IxA} - \text{Strategy Profile of players}$$

$$\prod^{\alpha}(m) \geq \prod^{\alpha}(\overline{m}^{\alpha}, m^{-\alpha}) \forall \overline{m}^{\alpha} \in R_{+}^{I}$$

- When it exists?
  - The cost function is continuous, convex and strictly increasing
  - The benefit function is continuous, convcave and increasing
  - The externality function is continuous
  - Customer has two distinct competing firms and customer responds to marketing of both the firms, and whose values on the customer strictly increases
- Challenges
  - Uniqueness of nash equillibria – Competition and Anonymous valuation

Research Areas of Focus

- Social Network Node Analysis – Game Theory
  - Social Theory
  - Value Theory
- Network Formation – Mechanism Design
  - Many areas
- Applications – Game Theory and Mechanism Design
  - Data Mining + Game Theory
  - Many exciting applications

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Thank You