Checking Unwinding Conditions for Finite State Systems

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MAKS Framework of Heiko

Events. Visible, Confidential, Neither

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Trace: finite sequence of events

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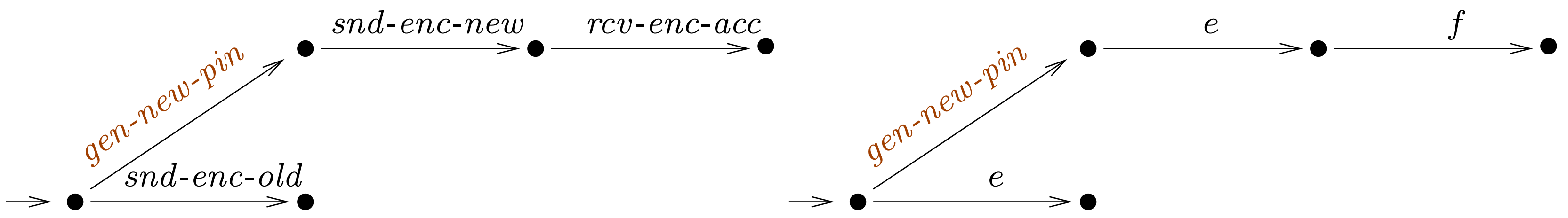
Non-Inference(NF)

$\forall \tau \in L \Rightarrow \exists \tau' \in L \;\; \tau' = \tau \upharpoonright_V$

An Example (1)



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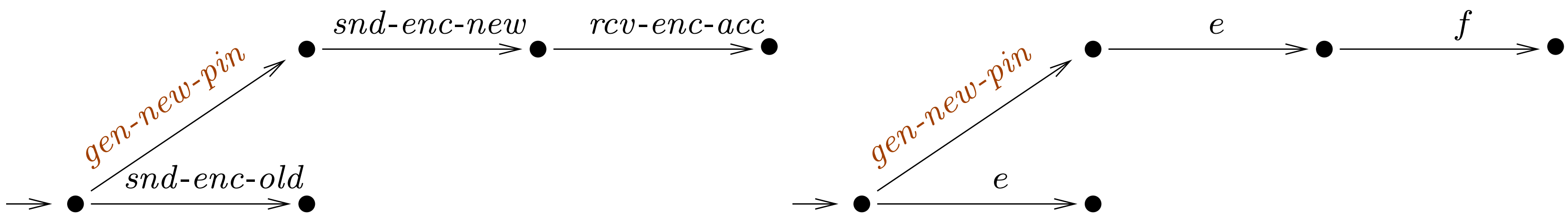

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$C = \{gen\text{-}new\text{-}pin\}$

$N = \phi$

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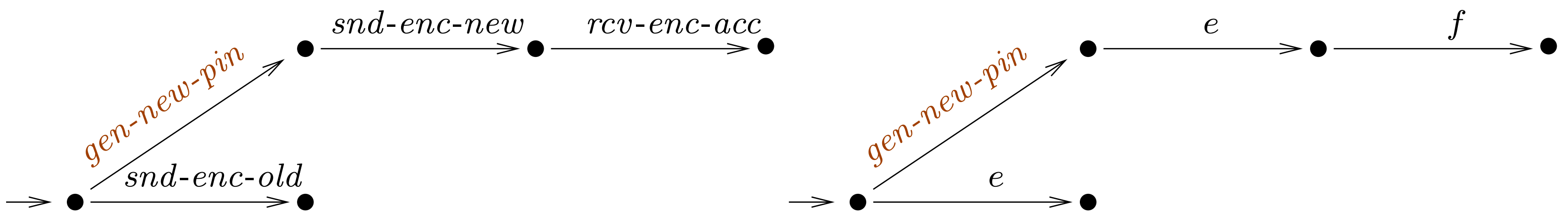

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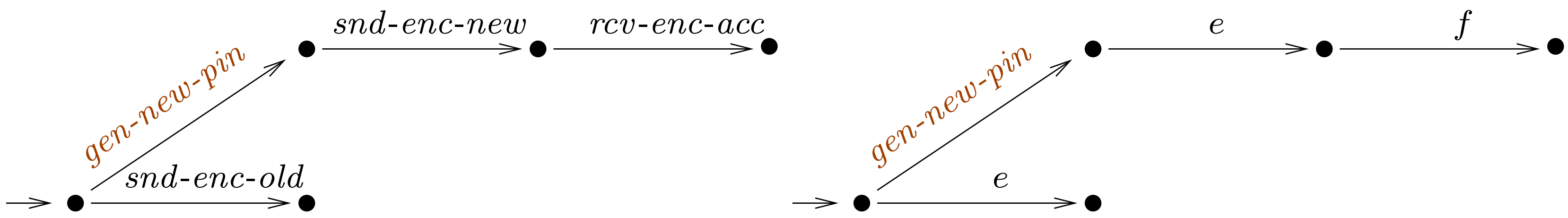

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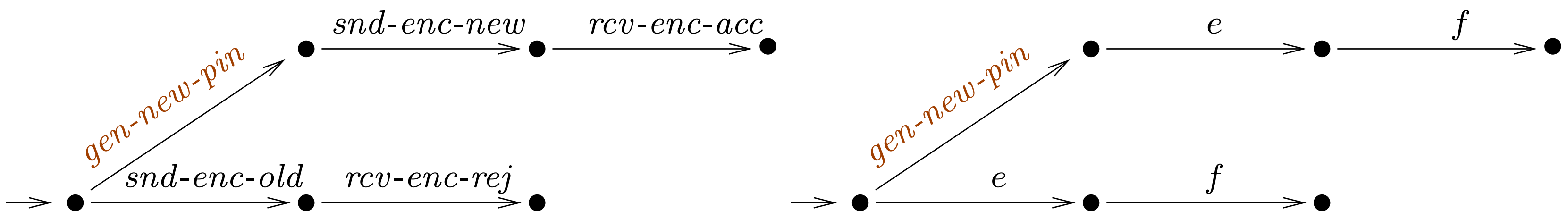
Confidentiality compromised. Noninference fails

An Example (2)



gen-new-pin
snd-enc-new
rcv-enc-acc
snd-enc-old
rcv-enc-rej
gen-new-pin
e
f
e
f

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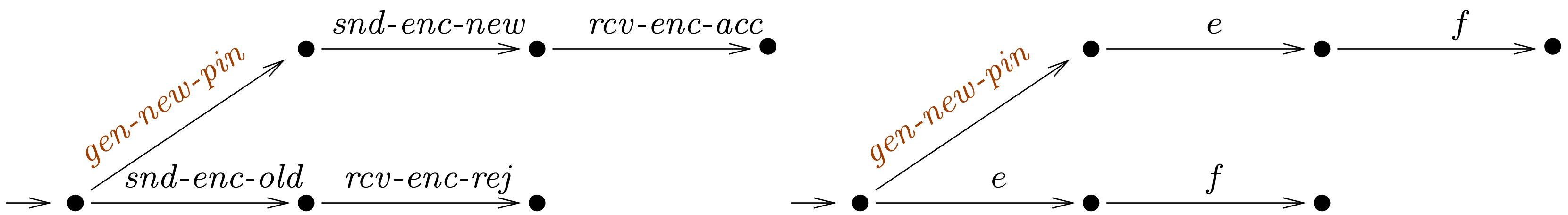

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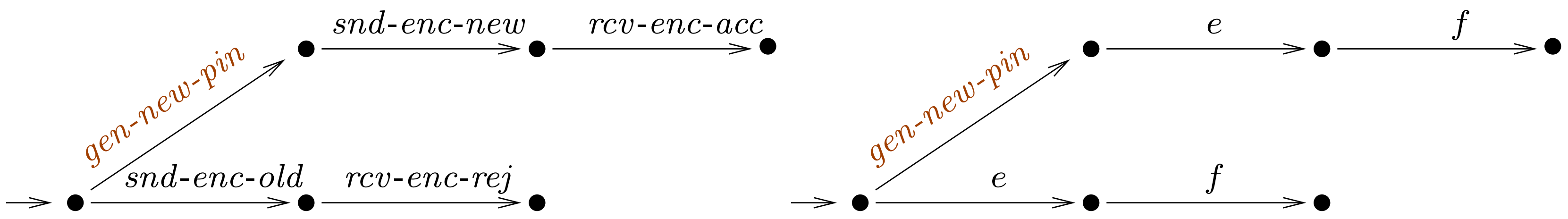

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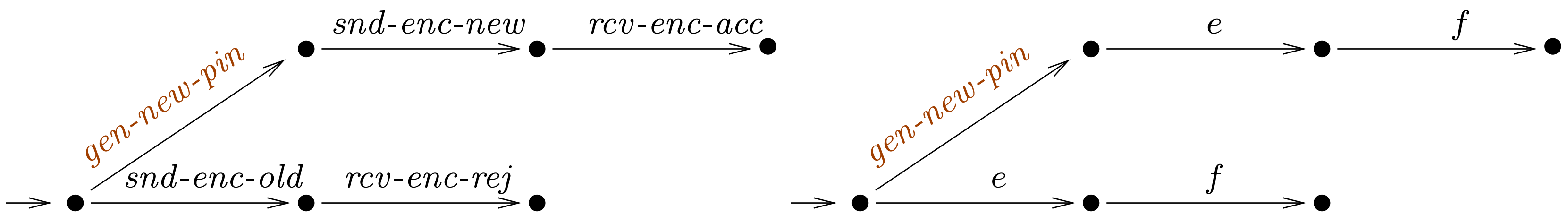

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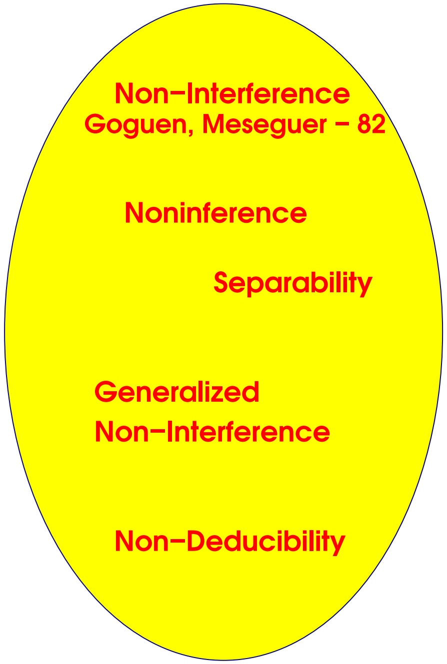
Confidentiality maintained. Noninference holds

Information Flow Properties



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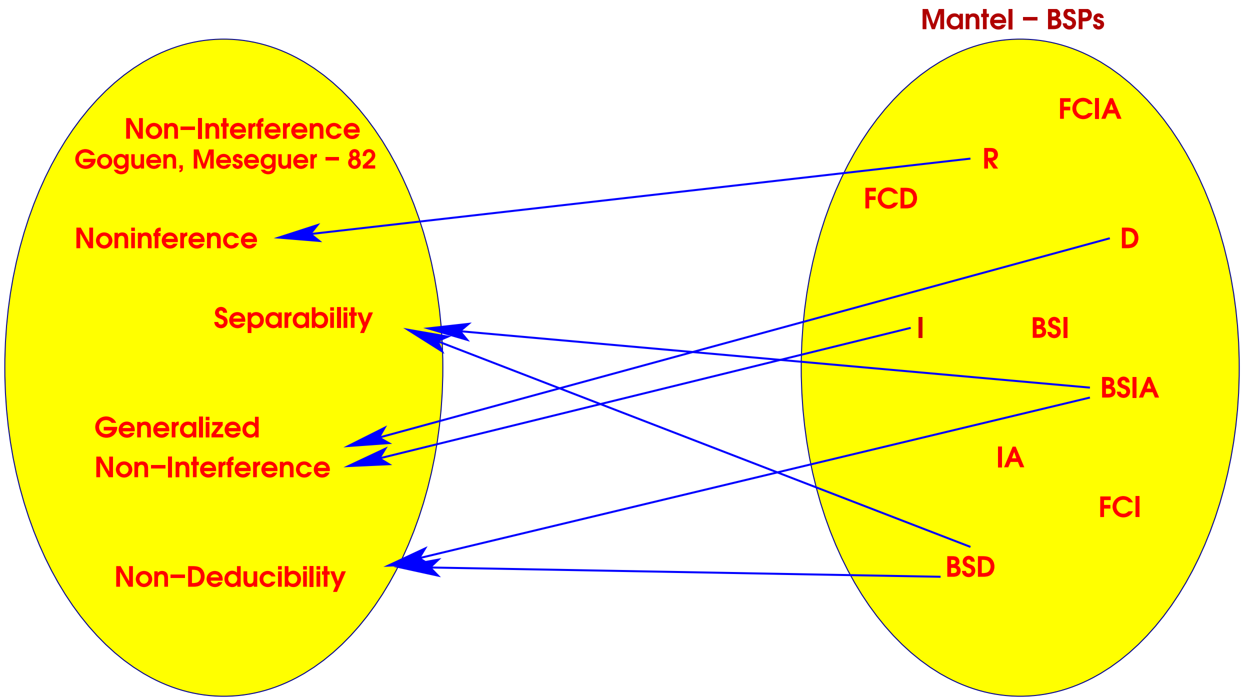

Basic Security Predicates (BSPs)

Trace based information flow properties in BSPs

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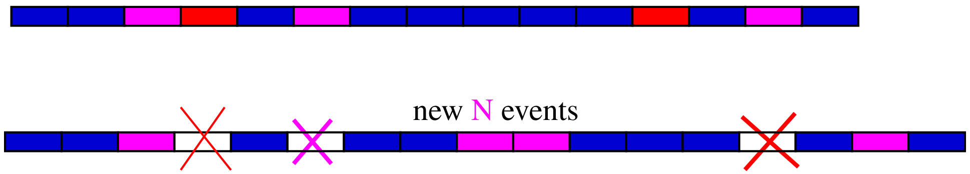
BSP Removal (R)



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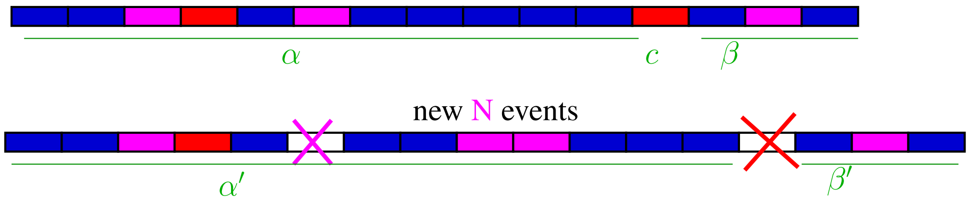
BSP Deletion (D)



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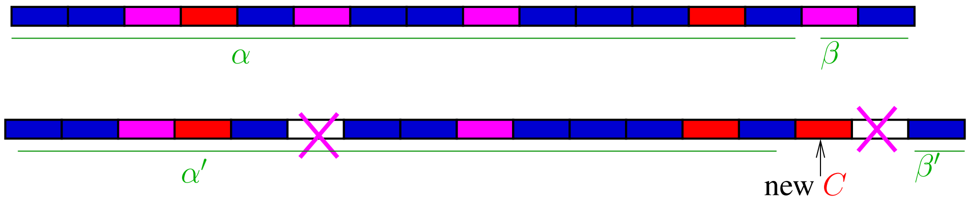

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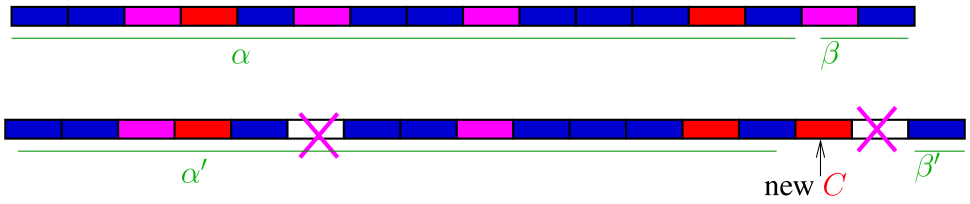

Generalized Non-Interference - I and D

Noninference - R

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Properties of sets of traces, Classical Model Checking techniques (Temporal Logic etc) cannot be used

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Examples

- L satisfies Removal R iff $L\upharpoonright_V \subseteq_N L$.
- L satisfies Deletion D iff *l-del*$(L) \subseteq_N L$.

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Regularity Preserving: Algorithm to construct automata for $op(L)$, given an automata for L

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Running time: Exponential in the size of the system

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Σ-labelled transition system $\mathcal{T} = (Q, s, \longrightarrow)$

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$fcrf, fcrb, lrbe, fcrbe$

Verification using Unwinding

Heiko's results

$\mathcal{T}$ satisfies

- BSD if there exists an unwinding relation $\ltimes$ such that $\mathcal{T}$ satisfies lrf w.r.t. $\ltimes$
- BSI if ... $\mathcal{T}$ satisfies lrb w.r.t. $\ltimes$
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Sufficient Conditions

Finitely many relations if finite states

Observations on Unwinding

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Similarly for lrb,...

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Well known Algorithms: {HHK}

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Naive Algorithm: Computing Maximal Simulation Relation

Input: $\mathcal{T}$, a finite state LTS

Output: $\prec_{\mathcal{T}}$, the maximal simulation relation for $\mathcal{T}$

for $p \in Q$

$\quad sim(p) = \{q \in Q \mid$ for all e enabled at p, e is also enabled at $q\}$

while there are states p, q, r and $e \in \Sigma$ such that

$r \in post_e(p),\ q \in sim(p)$ and $post_e(q) \cap sim(r) = \phi$ {

$\quad sim(p) = sim(p) \setminus \{q\}$

}

$\prec_{\mathcal{T}} = \bigcup_{q \in Q}\{\{q\} \times sim(q)\}$

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Proof follows due to the construction of $\mathcal{T}_V$

Unwinding as a Simulation Relation (2)

Checking Unwinding Conditions

1. Construct $\mathcal{T}_V$ from $\mathcal{T}$
2. Compute the maximal simulation relation $\prec_{\mathcal{T}_V}$ using standard algorithms
3. Check the unwinding conditions (lrf, ...) w.r.t. $\prec_{\mathcal{T}_V}$

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Feasible way to check, though not complete

Thank You